**9. ALGEBRA**

**Solution Exercise – Easy**

1. (a) : Let *f*(*x*) = 2*x*3 − *ax*2 − (2*a* − 3) *x* + 2

If (*x* + 1) is a factor of the above expression, then *f*(−1) = 0

 *f*(−1) = 2(−1)3 − *a*(−1)2 − (2*a* − 3) × (−1) + 2 = 0

 −2 − *a* + 2*a* − 3 + 2 = 0

 *a* − 3 = 0

 *a* = 3

***Solutions for questions 2 and 3:***

For no solution.



For unique solution.



For ∞ solutions.



where *a*1, *a*2, *b*1, *b*2, *c*1, *c*2 are the coefficient.

2. (d)

3. (b)

4. (a) : *k*2 + 2*k* − 3*k* − 6 + *k*2 + 3*k*

− 4*k* − 12 = 2*k*2 − 5

− 2*k* − 18 = − 5*k*

*k* = 

5. (c) : 

1010*x* = 10100

*x* = 10

6. (b) : Let *x* years be Ram's present age.

2*x* − 3 (*x* − 4) = *x*

2*x* − 3*x* + 12 = *x*

− *x* + 12 = *x*

2*x* = 12

*x* = 6

Present age of Ram = 6 years.

7. (c) : Let the number be *x*.





*x* = 48

8. (d) : Let = *u*, = *v*

3*u* + 2*v* = 3 ..... (1)

2*u* + 3*v* =  ..... (2)

Multiplying (1) by 2 and (2) by 3,

6*u* + 4*v* = 6 ..... (3)

6*u* + 9*v* = 11 ..... (4)

Subtracting (3) from (4),

5*v* = 5

*v* = 1

*u* = 

Now, *u* = 

 *a + b* = 3

and *v* = = 1

*a − b* = 1

Solving for *x* and *y*,

*a + b* = 3

*a* *− b* = 1

*a* = 2

*b* = 1

9. (b) : Let Abhinav's age is *x* and sum of ages of his son is *y*.

So, *x* = 3*y* ..... (1)

*x* + 5 = 2(*y* + 5 + 5) ..... (2)

From (1) and (2),

3*y* + 5 = 2*y* + 20

*y* = 15

*x* = 45

10. (d) : Let their income is in ratio 4*x* : 5*x*

We know,

Income = Expense + Saving

Income − Saving = Expense

Using this,



12*x* − 1500 = 10*x* − 1000

2*x* = 500

*x* = 250

4*x* = 1000

5*x* = 1250

11. (a) : Let wage of one women is *x* and wage of one boy is *y*.

8*x* + 6*y* = 41 ..... (1)

4*x* − 5*y* = 4.5 ..... (2)

Multiply (2) by 2 and subtracting from (1),

8*x* + 6*y* = 41

8*x* − 10*y* = 9

16*y* = 32

*y* = 2

Putting the value of *y* is equation (1),

8*x* + 6(2) = 41

8*x* = 29

12. (b) : Demand  4*q* + 7*r* = 17 ..... (1)

Supply  *r* =  ..... (2)

Multiply equation (2) by 12,

12*r* − 4*q* = 21

Solving equation (1) & (2),

7*r* + 4*q* = 17

12*r* − 4*q* = 21

19*r* = 38

*r* = 2

*q* = 

13. (c) : Let the digit in the unit's place be '*x*' and the digit in the ten's place be '*y*'.

The original number = 10*y* + *x*.

Number obtained by interchanging the digits = 10*x* + *y*.

Now, 10*y* + *x* = 4(*x* + *y*)

3*x* – 6*y* = 0 ..... (1)

Also, (10*x* + *y*) – (10*y* + *x*) = 27.

 10*x* + *y* – 10*y* – *x* = 27  9*x* – 9*y* = 27

 *x* – *y* = 3 ..... (2)

Multiplying (2) by 3,

3*x* – 3*y* = 9 ..... (3)

Subtracting (3) from (1),

–3*y* = –9  *y* = 3

Substituting the value of *y* in equation (2),

*x* – 3 = 3  *x* = 6

The number is 10 × 3 + 6 = 36.

14. (c) : Let '*x*' tickets be sold at Rs. 70 and '*y*' tickets be sold at Rs. 50

*x* + *y* = 77 ..... (1)

70*x* + 50*y* = 4890

 7*x* + 5*y* = 489 ..... (2)

Multiplying equation (1) by 7

7*x* + 7*y* = 539 ..... (3)

Subtracting equation (2) from (3),

2*y* = 50  *y* = 25

Hence, 25 tickets were sold at Rs. 50

15. (c) : *x*2 + 7*x* + 12

Sum of roots

*a + b* = − 7

*ab* = 12

roots are − 3 and − 4,

(*a + b*)2 = ((− 3) + (− 4))2 = 49

(*a − b*)2 = (4 − 3)2 = 1

Sum of roots for new equation

(*a + b*)2 + (*a − b*)2 = 49 + 1

= 50

Product of roots of new equation

(*a + b*)2 (*a* − *b*)2 = (49) × (1) = 49

So, our new equation is,

*x*2 − 50*x* + 49 = 0

16. (c) : 4*y* − 3 × 2*y* + 2 + 25 = 0

(2*y*)2 − 3 × 22 × 2*y* + 25 = 0

(2*y*)2 − 12 × 2*y* + 25 = 0

Put 2*y* = *u*

*u*2 − 12*u* + 32 = 0

*u*2 − 8*u* − 4*u* + 32 = 0

*u* (*u* − 8) − 4 (*u* − 8) = 0

*u* = 8, *u* = 4

2*y* = 8 or 2*y* = 4

*y* = 3 or *y* = 2

17. (d) : As *m* and *n* are the roots of the equation

*x*2 − *bx* + *c* = 0

then, *m + n* = −(−*b*) = *b*

*m* × *n* = *c*

Now, 

= 

= 

Product of roots

=

So, required equation is,



*cx*2 − (*b*2 − 2*c*)*x* + 1 = 0

18. (d) : Let root be *a* and *b*.

So, sum of roots

*a + b* =  = − 4

2*a* = − 4

*a* = − 2 (roots are equal)

Now, product of roots

*a* × *b* = 

− 2 × − 2 = 

*k* = − 2

19. (b) : Let *x* = 

*x* = 4 + 

*x*2 = 4*x* + 1

*x*2 − 4*x* − 1 = 0

*x*2 − 4*x* + 4 − 4 − 1 = 0

(*x* − 2)2 = 5

*x* = ± + 2

*x* =

2 − will get discard, since there is no negative term in equation.

20. (b) : If *m* and *n* are the roots. So product of roots = *m* × *n* = 

Sum of roots 1 (*m + n*) = 

Squaring both side

(*m + n*)2 = 4

*m*2 + *n*2 + 2*mn* = 4

*m*2 + *n*2 = 4 − 2 × 

*m*2 + *n*2 = 7

21. (a) : Let α and β are the roots of equation *px*2 + *qx + r* = 0



Now, sum of roots

 ..... (1)

Product of roots

 ..... (2)

Dividing (1) by (2),



=

Squaring both side





Given 







22. (c) : Let roots are *a* and *b*.

Sum of roots

*a* + *b* = (*k* + 4)

2*a* = *k* + 4

*a* = ..... (1)

Product of roots

*a*2 = 2*k* + 5 ..... (2)

Squaring equation (1) and substituting in (2),



*k*2 + 16 + 8*k* = 8*k* + 20

*k*2 = 4

*k* = ±2

23. (b) : Let roots of equation be *k* and *p*.

So, sum of the roots

*k + p* =  ..... (1)

Product of roots

*k* × *p* = 

*p* =  or *k* = 0

On putting the value of *p* is equation (1),

*k* = 

*k* = − 2

24. (d) : *x*2 − *x* − 1 = 0

*x*2 − *x* + 



*x* = 



Value of 

=

= 

= 







25. (b) : Let roots of equation be *b* and .

Product of the roots



*k* = 5

26. (d) : Le the root of the equation be *x* and *x* + 4 sum of the roots.

*x* + *x* + 4 = 8

2*x* + 4

*x* = 2

So, roots are 2, 6.

Product of roots

(*x*) (*x* + 4) = *m*

(2)(6) = *m*

*m* = 12

27. (d) : 





6*x*2 + 6*x*2 + 12*x* + 6 = 13*x*2 + 13*x*

6 = *x*2 + *x*

*x*2 + *x* − 6 = 0

*x*2 + 3*x* − 2*x* − 6 = 0

*x*(*x* + 3) − 2 (*x* + 3) = 0

(*x* = 2, − 3)

28. (b) : *x*2 + 9*x* + 18 = 6 − 4*x*

*x*2 + 13*x* + 12 = 0

*x*2 + 12*x* + *x* + 12 = 0

*x*(*x* + 12) + 1(*x* + 12) = 0

(*x* + 1) (*x* + 12)

(*x* = − 1, − 12)

29. (c) : 





49*x*2 + 231 = 4*x*2 + 276

45*x*2 = 45

*x* = ± 1

30. (c) : Let the parts be *x* and 25 − *x*.

By the question 

or 

or 150 = 25*x* − *x*2

*x*2 − 15*x* − 10*x* + 150 = 0

*x*(*x* − 15) − 10(*x* − 15) = 0

(*x* − 10) (*x* − 15) = 0

*x* = 10, 15

31. (a) : Let suppose the number is *x*.

2*x*2 − 5*x* = 3

2*x*2 − 5*x* − 3 = 0

2*x*2 − 6*x* + *x* − 3 = 0

2*x*(*x* − 3) + 1(*x* − 3) = 0

(2*x* + 1)(*x* − 3) = 0

*x* = 

32. (b) : *x*3 − 6*x*2 + 11*x* − 6 = 0

By hit and trail put *x* = 1

1 − 6 + 11 − 6 = 0

0 = 0

So, (*x* − 1) is one of the root.

Now divide whole equation with (*x* − 1),



Equation can be written as:

(*x* − 1) (*x*2 − 5*x* + 6)

(*x* − 1) (*x*2 − 3*x* − 2*x* + 6)

(*x* − 1) [*x*(*x* − 3) − 2 (*x* − 3)]

(*x* − 1) (*x* − 2) (*x* − 3)

*x* = 1, 2, 3 are the solutions.

33. (a) : 4*x*3 + 8*x*2 − *x* − 2 = 0

It cannot become 0 by 1 or − 1, therefore

By hit and trail put *x* = − 2

4(− 2)3 + 8(− 2)2 − (− 2) − 2 = 0

− 32 + 32 + 2 − 2 = 0

0 = 0

So, one factor is *x* + 2

Now, divide the equation with (*x* + 2),



So, equation can be written as

(*x* + 2) (4*x*2 − 1) = 0

(*x* + 2) (2*x* + 1) (2*x* − 1) = 0

*x* = 

So, value of (2*x* + 3)

*x* = − 2 , 2(− 2) + 3 = − 1

*x* = , 2 + 3 = 4

*x* = , 2 + 3 = 2

So, options (a) is right answer.

34. (a) : *a* = 1 , b = 3, *c* = 

α . β = 

= 

α = β

35. (a) : 2*x*2 + (*k* + 1) *x* + 8 = 0.

*b*2 − 4*ac*

= (*k* + 1)2 − 4 × 2 × 8

= *k*2 + 2*k* + 1 − 64

= *k*2 + 2*k* − 63 = 0

*k*2 + 2*k* − 63 = 0

 *k* = 7, − 9.

36. (a) : For the roots to be reciprocal *c* = *a* of another equation.

of one equation

Here, *c = a = p* (same).

37. (b) : Apply the formula (4 − 2) (− 10 − 4*c*) = (− 5 − *c*)2

 *c* = − 15 or − 3

38. (b) : α + β = − 1, αβ = 1, α2 + β2 = (− 1)2 − 2 = − 1

α2 β2 = 1.

Hence, *X* 2 + *X* + 1 = 0 is the required equation.

39. (b) : Roots are same but opposite in sign. So, if α, β be the roots of the equation the α + β = .

αβ =  for other equation − α + (− β) =.

− β =  use these two conditions on the given equations and the equation can be found out.

40. (b) : From the first error, sum is not affected  Sum = − 7

From the second error, product is not affected Product = 12

41. (a) : For equation *x*2 − 23*x* + 42 = 0, *S* = (α + β) = 23, *P* = αβ = 42

The required equation has the roots as . Therefore, the sum of the roots of the given equation is

*s* =,

*P* =  = 5.6 × 10− 4. So, required equation is *x*2 − 2.6*x* + 5.6 × 10 − 4 = 0.

42. (b) : Sum of roots = *a* + *b* = 6

Product of roots = *a* × *b* = 6

Now, *a*2 + *b*2 = (*a* + *b*)2 − 2*ab*

= 36 − 12 = 24

43. (c) : Sum of roots = −13

Product or roots = −30

 Equation

*x*2 − *x* (sum of roots) + product of roots = 0

*x*2 + 13*x* − 30 = 0

44. (d) : (*x* − 3) (2*x* + 1) = 0

Then, (*x* − 3) = 0

 *x* = 3

and (2*x* + ****) = 0

If *x* = 3

Then (2*x* + 1) = 2 × 3 + 1 = 7

Similarly, 

 Possible values of (2*x* + 1) are 0 and 7.

45. (d) : If we write the given equation in the conventional form,

*i.e*. *ax*2 + *bx* + *c*,then we have *a* = 1, *b* = – (*A* – 3), *i.e*. (3 – *A*) and *c* = –(*A* – 2), *i.e*. (2 – *A*). Let the roots of this equation be α and β.

So the sum of the squares of the roots

= α2 + β2 = (α + β)2 – 2αβ.

Now (α + β) = Sum of the roots = 

=  = (*A* – 3), and

αβ = Product of the roots = 

=  = (2 – *A*).

Hence, α2 + β2 = (*A* – 3)2 – 2(2 – *A*)

= *A*2 – 4*A* + 5 = 0.

Instead of finding the values of *A*, we put the values from the given options.

None of the answer choices matches this equation.

46. (d) : If the roots are *a* and *a*2, then product of the roots = *a*3 = –8.

 *a* = –2. Hence, sum of the roots = *k* = –(*a* + *a*2)

= –(–2 + 4) = – 2.

47. (a) : If one root of *x*2 + *px* + 12 = 0 is 4, then 42 + 4*p* + 12 = 0, *i.e.* *p* = –7.

7*x* + *q* = 0 has equal roots.

 If the roots are α each,

.

48. (c) : 7*x* – 5 ≥ 5*x* + 9  2*x* ≥ 14

3*x* – 19 ≤ 8  3*x* ≤ 27

 2*x* ≥ 14 and 3*x* ≤ 27

 *x* ≥ 7 & *x* ≤ 9.

Combining both we get the generalized solution for *x*.

7 ≤ *x* ≤ 9.

Hence, option (c) is correct.

49. (d) : | *X* + 3| > 2 is equivalent to *X* + 3 > 2

 *X* ≥ – 3 or – (*X* + 3) > 2

 *X* < – 3

 *X*  > – 1, *X* ≥ – 3 or *X*  < – 5, *X*  < – 3.

Hence, *X* > – 1 or *X* < – 5.

50. (c) : (*X* – 4) < ± 4  0 < *X* < 8 taking once the positive and once the negative sign.

51. (d) : The equation is equivalent to : *X* 2 + 5*X* + 6 = 0, *X* ≥ 0 or *X* 2 – 5*X* + 6 = 0, *X* < 0.

*X* = – 3 or – 2 but these values do not satisfy *X* ≥ 0.

Similarly from the second equation we have no root.

52. (c) : Let us remove the outermost modulus, then the equation reduces to | *x* – 1 | – 6 | – 10 = ± 0.

 | *x* – 1 | – 6 | = 10

Now, again repeating the same step.

We have | *x* – 1 | – 6 = 10, | *x* – 1 | – 6 = – 10

 | *x* – 1 | = 16, | *x* – 1 | = – 4

(This case is not possible, as modulus cannot have negative values).

Again, removing the modulus,

*x* – 1 = 16, *x* – 1 = – 16

*x* = 17, *x* = – 15

 *x* = 17 and – 15 are the possible values.

53. (d) : Using AM ≥ GM

 

54. (b) : *T*5 = *a* + (*n* − 1) *d*

2 = −14 + 4*d*



 

 

 80 = −28*n* + 4*n*2 − 4*n*

 4*n*2 − 32*n* − 80 = 0

 *n*2 − 8*n* − 20 = 0

 (*n* − 10) (*n* + 2) = 0

 *n* = 10 (Since *n* cannot be negative)

55. (a) : *n*th term of a G.P. = *arn* − 1

 8th term = 5 × (2)8 − 1 = 5 × (2)7 = 5 × 128 = 640

56. (a) : 

=  [2 × 4 + (40 × 1) 4]

= 20 [8 + 39 × 4]

= 20 [8 + 156]

= 3280

57. (c) : We do not need to apply any formula in this case. The middle term of an AP is always the average of all the terms. Hence, if we multiply the middle term by the number of terms, we should get the sum of all the terms of that AP. In our problem, we have to find the sum of first 7 terms and we have been given the 4th term (which is the middle term). Hence the required answer is 8 × 7 = 56.

58. (c) : *t*1 = *a* (say)

common difference = *d* (say)

 (*a* + 4*d*) + (*a* + 6*d*) = 16 *a* + 5*d* = 1 ..... (1)

Again, *a* + 2*d* + *a* + 12*d* = 28

*a* + 7*d* = 14 ..... (2)

Subtracting (1) from (2) we get, 2*d* = 6 *d* = 3

 From (1), *a* + 5*d* = 8

 8 – 5 × 3 = – 7

 *t*6 = *a* + 5*d* = – 7 + 15 = 8

Hence, option (c) is correct.

59. (c) : 



= 

 A.M. – G.M. = 156 – 56 = 100

Hence, option (c) is correct.

60. (c) : Since both 2 and 1 are positive, (2 # 1) = 2 + 1 = 3.

(12)= (1 × 2)1 + 2 = 23 = 8.

Thus, the given expression is equal to.

61. (a) : Let us first simplify the numerator. Since 1 is positive,

(1 # 1) is 1 + 1 = 2 which again is positive. Then

(1 # 1) # 2 = 2 # 2 = 2 + 2 = 4

Now, note that log10 0.1 = log10 10–1 = –1

Then 101.3log10 0.1= 101.3 × (–1) is negative.

So 101.3log10 0.1 = 1

Hence, the numerator is equal to 4 – 1 = 3

Since 1 × 2 = 2 is positive, (12) = (1 × 2)1 + 2 = 23 = 8.

So, the denominator = 8. Hence, the answer is.

62. (b) : The best possible way to solve this is to check each of the given answer choices. In options (a), (c) and (d), either both *X* and *Y* are positive or both *X* and *Y* are negative. Since we have (–*Y*) in the numerator of our expression and (–*X*) in the denominator, *X* and *Y* will never be both positive and neither will *XY* be positive because both the numerator and the denominator of our expression will be 1 and the value will always be 1. Hence, the only possible answer choice is (b).

63. (b) : Let  = *y*.

So, *f0 g*(*x*) = *f0* (*y*) = 2*y* − 3.

Substituting, we get

log(*x*) = 

= *x* − 3 − 3

= *x*

This can also be written as, and this we see is nothing 2 *g*0 *f*(*x*).

64. (c) : 

65. (c) : *F*(1) = −(1)3 + 5 = 4

*F*(9) = (9)3 + 5 = 734

*F*(−1) = −7

*F*(0) = −(0)3 + 5 = 5

*F*(1) + *F*(9) + *F*(−1) + *F*(0) = 4 + 734 − 7 + 5 = 736

Hence, option (c) is correct.

66. (c) : *f*(*x* + 1) = *x*2 − 3*x* + 2 = (*x* − 2) (*x* − 1) ..... (1)

Substituting (*x* − 1) in place of *x* in (1),

*f*(*x*) = (*x* − 3) (*x* − 2) = *x*2 − 5*x* + 6.

Hence, option (c) is correct.

67. (b) : *f*(*x* + 1) = *x*2 − 3*x* + 2. Putting *x* = 3 we get, *f*(4) = 9 − 9 + 2 = 2.

Hence, option (b) is correct.

68. (a) : *f*(*x* + 1) = *x*2 − 3*x* + 2 = (*x* − 2) (*x* − 1) ..... (1)

Substituting (*x* + 1) in place of *x* in (1),

*f*(*x* + 2) = (*x* − 1). (*x*) = *x*2 − *x*.

Hence, option (a) is correct.

69. (d) : 

 At *x* = 1, the denominator becomes zero. Hence, the function *f*(*x*) is undefined at *x* = 1.

Hence, option (d) is correct.

70. (a) : *f*(*x*) = 2*x* + 7 = y (say)



Hence, option (a) is correct.

**Solution Exercise – Medium**

1. (d) : 

2. (a) : *x*3 − 19*x* + 30

We put the values from the options to solve the question.

(*x* − 2) is factor of the above expression, because for *x* = 2(2)3 − 19 × (2) + 30 = 0

3. (a) : 

= 

= 

4. (b) : Let 

 *x* = *K*, *y* = 3*K*, *z* = 5*K*

 

= 

5. (c) : Here *x*, *y*, *z* are distinct positive real number



= 

=  [We know that

 if a and b are distinct numbers

> 2 + 2 + 2

> 6

6. (c) : 5*x* + 19*y* = 64

We see that if *y* =1, we get an integer solution for *x* = 9, now if y changes (increases or decreases) by 5*x* will change (decrease or increase) by 19. Looking at the options, if *x* = 256, we get *y* = 64. Using these values we see options (a), (b) and (d) are eliminated and also that these exists a solution for 250 < *x* ≤ 300.

7. (d) : 4*x* – 17*y* = 1. And given that 1000 ≥ *x*

Hence, we can say that 17*y* + 1 ≤ 4000

*i.e*., *y* ≤ 235

Further also note that every 4th value of *y* e.g., 3, 7, 11, ..... will give an integer value of *x*.

So, number of values of *y* =  integral values.

8. (a) : Let the cost of the turban be *T*. Hence, total payment for one year = Rs. 90 + *T*. So the payment for 9 months should be

 But this is equal to (65 + *T*). Equating the two, we get *T* = Rs. 10.

9. (a) : According to this





= 2

But putting *x* = 2 in the denominator of  we get the

denominator as 0 which is not defined, hence there are no real values.

Hence, option (a) is correct.

10. (d) : *x*2 + 6*x* + *y*2 = 4

or *x*2 + 6*x* + 9 + *y*2 = 13

or (*x* + 3)2 + *y*2 = 13

Let *y* = 0; No integer value of *x*,

*y* = ± 1; No integer value of *x*,

*y* = ± 2; (*x* + 3)2 = 9  *x* = 0, –6

*y* = ± 3; (*x* + 3)2 = 4  *x* = –1, –5.

So, total 8 sets of integers are possible,

*i.e*., (0, 2), (0, –2), (–6, 2), (–6, –2), (–1, 3), (–1, –3), (–5, 3), (–5, –3)

11. (b) : Given, 

Now, 

= 

= 

12. (c) : 2*x*2 − 7*xy* + 3*y*2 = 0



 

= 

 

13. (c) : 

 

 2*x* +  = 35 + 6

 (2*x* − 41) = − 

 (2*x* − 41)2 = 4(*x*2 − 6*x* − 40)

 4*x*2 + 1681 − 164*x* = 4*x*2 − 24*x* − 160

 140*x* = 1841

 

14. (c) : Roots of a quadratic equation

*ax*2 + *bx* + *c* = 0 are real if *b*2 − 4*ac* ≥ 0

Option (a): 3*x*2 − 4*x* + 5 = 0

*b*2 − 4*ac* = (−4)2 − 4 (3) (5) = −44 < 0

Hence, roots are not real.

Option (b): *x*2 + *x* + 4 = 0

*b*2 − 4*ac* = (1)2 − 4 (1) (4) = 1 − 16 = −15 < 0

Hence, roots are not real.

Option (c): (*x* − 1) (2*x* − 5) = 0

 *x* = 1 and *x* = 

Hence, roots real are

Option (d): 2*x*2 − 3*x* + 4 = 0

*b*2 − 4*ac* = (−3)2 − 4 (2) (4) = 9 − 32 = −23 < 0

Hence, roots are not real.

15. (d) : Clearly, equations given by options (a) and (c) are not quadratic equation.

Now, testing option (b), (*x* − 1) (*x* + 4) = *x*2 + 1

 *x*2 + 4*x* − *x* − 4 = *x*2 + 1

 3*x* − 5 = 0

It is not a quadratic equation.

Option (d), (2*x* + 1) (3*x* − 4) = 2*x*2 + 3

 6*x*2 − 8*x* + 3*x* − 4 = 2*x*2 + 3

 4*x*2 − 5*x* − 7 = 0

and this is a quadratic equation.

16. (a) : 

= 

= 

17. (d) : 

 25 − *x*2 = (*x* − 1)2

 25 − *x*2 = *x*2 + 1 − 2*x*

 2*x*2 − 2*x* − 24 = 0

 *x*2 − *x* − 12 = 0

 (*x* − 4) (*x* + 3) = 0

 *x* = 4, *x* = −3

 *x* = 4 ( *x* = −3 is not satisfy the given equation)

18. (b) : 



 

19. (a) : 

 



 

−8 ≤ *x* ≤ 1

20. (d) : *p* + *q* = α –2 and *pq* = –α – 1

(*p* + *q*)2 = *p*2 + *q*2 + 2*pq*,

Thus (α –2)2 = *p*2 + *q*2 + 2(–α – 1)

*p*2 + *q*2 = α2 – 4α + 4 + 2α + 2

*p*2 + *q*2 = α2 – 2α + 6

*p*2 + *q*2 = α2 – 2α + 1 + 5

*p*2 + *q*2 = (α – 1)2 + 5

Thus, minimum value of *p*2 + *q*2 is 5.

21. (b) : *ax*2 + *bx* + 1 = 0

For real roots

*b*2 − 4*ac* ≥ 0

 *b*2 − 4*a*(1) ≥ 0

 *b*2 ≥ 4*a*

For *a* = 1, 4*a* = 4,  *b* = 2, 3, 4

*a* = 2, 4*a* = 8,  *b* = 3, 4

*a* = 3, 4*a* = 12,  *b* = 4

*a* = 4, 4*a* = 16, *b* = 4

 Number of equations possible = 7.

22. (b) : Use the choices. If *b* = 1, then the factors are (*x* – *a*)

(*x*2 + 1). This cannot yield 3 real roots.

23. (a) : We know that the sum of the roots = 

Hence, *x*1 + *x*2 = 2. Now we have two equations, viz. *x*1 + *x*2 = 2 and 7*x*2 – 4*x*1 = 47. Solving these two equations, we get *x*1 = –3 and *x*2 = 5. Since it does not satisfy options (b) and (c), we will verify it for option (a). The product of roots = (–3) × 5 = –15,

in our case is c. Hence, c = –15.

Alternative **Method:**

Put values of *x*1, *x*2 in equation (2). They not match.

So, put *c* = –15 in equation (1) to get the roots of equation.

After finding the roots of equation (1), check whether they satisfy equation (2) or not. The roots (5, –3) satisfy equation (2) so answer is (a).

24. (a) : If we simplify the expression *x*2 – 3*x* + 2 > 0, we get (*x* – 1)(*x* – 2) > 0. For this product to be greater than zero, either both the factors should be greater than zero or both of them should be less than zero.

Therefore, (*x* – 1) > 0 and (*x* – 2) > 0 or (*x* – 1) < 0 and (*x* – 2) < 0.

Hence, *x* > 1 and *x* > 2 or *x* < 1 and *x* < 2. If we were to club the ranges, we would get either *x* > 2 or *x* < 1. So for any value of *x* equal to or between 1 and 2, the above equation does not follow.

25. (c) : 3*m*2 – 21*m* + 30 < 0

or *m*2 – 7*m* + 10 < 0, or *m*2 – 5*m* – 2*m* + 10 < 0

or *m* (*m* – 5) – 2 (*m* – 5) < 0

or (*m* – 2) (*m* – 5) < 0

Case **I:** *m* – 2 > 0 and *m* – 5 < 0

 *m* > 2 and *m* < 5  2 < *m* < 5

Case **II:** *m* – 2 < 0 and *m* – 5 > 0 *m* < 2 and *m* > 5

nothing common.

Hence, 2 < *m* < 5

26. (b) : *x*3 – *ax*2 + *bx* + *c* = 0

Let the roots of the above cubic equation be

(α – 1), α, (α + 1)

 α (α – 1) + α (α + 1) + (α + 1) (α –1) = *b*

 α2 – α + α2 + α + α2 – 1 = *b*

 3α2 – 1 = *b*

Thus, the minimum possible value of ‘*b*’ will be equal

to – 1 and this value is attained at α = 0.

Hence, option (b) is the correct answer.

27. (b) : Given that *f*(*x*) = *ax*2 + *bx* + *c*

Also, *f*(5) = –3*f*(2) *f*(5) + 3*f*(2) = 0

(25*a* + 5*b* + *c*) + 3(4*a* + 2*b* + *c*) = 0

37*a* + 11*b* + 4*c* = 0 ..... (1)

Also, as 3 is a root of *f*(*x*) = 0, therefore *f*(3) = 0.

Therefore, 9*a* + 3*b* + *c* = 0 ..... (2)

Using equation (i) and (ii), we get that *a* = *b*

Therefore, *c* = –12*a*

*f*(*x*) = *a*(*x*2 + *x* –12) = *a*(*x* + 4) (*x* – 3)

Therefore, the other root of *f*(*x*) = 0 is –4.

Hence, option (b) is the correct choice.

28. (d) : *f*(*x*) = *a*(*x*2 + *x* –12)

The value of *a* + *b* + *c* cannot be uniquely determined.

Hence, option (d) is the correct choice.

29. (a) : Let the roots be *a* – *d*, *a*, *a* + *d*

Let us also compare the equation with *ax*3 + *bx*2 + *cx + d* = 0

The sum of the roots = 

 *a* – *d* + *a* + *a* + *d* = 

Product of the roots = 



 

 

 



[Note: Do not try to put the values of x from the given option as the given equation involves lengthy calculation.**]**

30. (b) : Since the roots are on the number line so *D* ≥ 0 *b*2 – 4*ac* ≥ 0

 *b*2 ≥ 8*a*

If *b* = 2 or 3 no value possible, *b* = 5, the *a* = 2 or 3 can satisfy the inequality

If *b* = 7, then *a* = 2, 3 or 5

Hence total number of equations possible = 5

31. (b) : *x*2 + 4 |*x*| – 4 = 0

If *x* > 0 = *x*2 + 4*x* – 4 = 0



But as *x* > 0 so *x* = 

If *x* < 0  *x*2 – 4*x* – 4 = 0



 *x* = 2 − 2 (As *x* < 0)



32. (d) : Let *y* denote the given expression

*y* = –(2*x*2 – 19*x* + 35) = –(2*x* – 5) (*x* – 7)

= (2*x* – 5) (7 – *x*) = 2 (7 – *x*)

For *y* to be positive

*i.e*.,  (*x* – 7) < 0   < *x* < 7

33. (c) : 





34. (c) : Since α, β are roots of the equations 2*x*2 – 3*x* – 6 = 0

α + β =  and α β = – 3

 α2 + β2 = (α + β)2 – 2α β = 

Now, (α2 + 2) + (β2 + 2) = α2 + β2 + 4 = 

(α2 + 2) + (β2 + 2) = α2 β2 + 2α2 + 2β2 + 4

= (– 3)2 + 2

So, the equation whose roots are α2 + 2 and β2 + 2 is:

*X* 2 – {(α2 + 2) + (β2 + 2)}*X* + (α2 + 2)(β2 + 2) = 0

or 

4*X* 2 – 49*X* + 118 = 0

35. (a) : Let α be the common root of the two equations. Then

2α2 + *k*α – 5 = 0 ..... (1)

and α2 – 3α – 4 = 0 ..... (2)

Solving these two equations by cross multiplication, we get



 α = −  (4*k* + 15) or α = −

 4*k*2 + 39*k* + 81 = 0

 (*k* + 3)(4*k* + 27) = 0.

Then, either *k* = − 3 or *k* = 

36. (d) : 13*x* + 1 < 2*z* and *z* + 3 = 5*y*2

13*x* + 1 < 2 (5*y*2 − 3)

13*x* + 1< 10*y*2 − 6

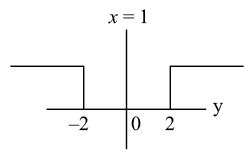
13*x* + 7 < 10*y*2 put *x* = 1

20 < 10*y*2 *y*2 > 2

*y* >  (*y*2 − 2) > 0

We see that none of the first three options satisfy these conditions.

Hence, option (d) is correct.



37. (d) : *x* > 5, *y* < –1

Use answer choices.

Take *x* = 6, *y* = –6. We see none of the statements

(1, 2 and 3) is true.

Hence, the correct option is (d).

38. (a) : Equation of quadratic equation is

*ax*2 + *bx* + *c* = 0

*x*2 + *bx* + *c* = 0

First roots are (4, 3)

Sum of the roots =  = − 7  *b* = 7.

Product of the roots =  = 12  *c* = 12.

 Equation formed *x*2 – 7*b* + 12 = 0 ..... (1)

Another boy gets the wrong roots (2, 3).

 Sum of the roots =  = −5  *b* = 5.

Product of the roots =  = 6  *c* = 6.

Equation formed *x*2 – 5*b* + 6 = 0 ..... (2)

*x*2 + *b′x* + *c*1 = 0

*b′* = 2 + 3

 *c* = 6

and *b* = 7 are the right coefficients.

Hence, *x*2 – 7*x* + 6 = 0

 *x*2 – 6*x* – *x* + 6 = 0

 *x*(*x* – 6) – 1(*x* – 6) = 0

 (*x* – 6)(*x* – 1) = 0

 *x* = 6, 1

Hence, the actual roots = (6, 1).

Alternate method:

Since, constant = 6[3 × 2] and

Coefficient of *x* = [–4*x* – 3*x*] = –7

Since quadratic equation is

*x*2 – (Sum of roots)*x* + Product of roots = 0 or

*x*2 – 7*x* + 6 = 0

Solving the equation (*x* – 6)(*x* – 1) = 0 or *x* = (6, 1).

39. (b) : Using options the answer is (b), because – *x* < –2 and

–2 < 2*y* –*x* < 2*y*.

40. (b) : There are two possibilities to this inequality.

Case **A:** *x* – 4 > 1 and 2*x*2 – 8*x* > 0, then the inequality will hold true.

*i.e*. *x* > 5 and 2*x*(*x* – 4) > 0  *x* > 5 and *x* does not lie between 0 and 4 *i.e*. (5, ∞)

Case **B:** 0 < *x* – 4 < 1 and 2*x*2 – 8*x* < 0, then again the inequality will hold true.

*i.e*., 4 < *x* < 5 and *x*(*x* – 4) < 0  4 < *x* < 5 and 0 < *x* < 4. This is an impossible situation.

Hence, the solution set is (5, ∞)

41. (c) : 43*x* – 2 > 1 or, 43*x* – 2 > 40

 3*x* – 2 > 0 

Hence, option (a) is correct.

42. (c) : Let *X* =  then, *X* = 

 *X* 2 = 6 + *X* *X* 2 – *X* – 6 =

 (*X* – 3) (*X* + 2) = 0

 either *X* = 3 or *X* = – 2.

But the given expression is +ve.

So, *X* = 3

43. (b) : |*a* − *b*| = |8| = 8

|*b* − *a*| = |−8| = 8

 |*a* − *b*| − |*b* − *a*| = 8 − 8 = 0

44. (b) : Given *a* − *b* + *c* < 1

and *a* + *b* + *c* > − 1

 − *a − b − c* < 1

 2*a* + 2*c* < 0

 *a* + *c* < 0

− 3*a* − 3*b* − 3*c* < 3 ..... (1)

Also, 9*a* + 3*b* + *c* < − 4

 6*a* − 2*c* < − 1 ..... (2)

From Eqs. (i) and (ii),

2*a* + 2*c* < 0

6*a* − 2*c* < −1

 8*a* < −1

 *a* < 

45. (b) : *x*2 + *y*2 = 0.1

|*x* – *y*|2 = *x*2 + *y*2 – 2*xy*

(0.2)2 = 0.1 – 2*xy*

or 2*xy* = 0.06 or *xy* = 0.03

Now |*x*| + |*y*| = 

|*x*| + |*y*| = 0.40

Hence, *x* = 0.3, *y* = 0.1 or vice versa.

46. (b) : Case **1:** If *x* < 2, then *y* = 2 – *x* + 2.5 – *x* + 3.6 – *x*

= 8.1 – 3*x*

This will be least if *x* is highest *i.e*. just less than 2.

In this case *y* will be just more than 2.1

Case **2:** If 2 ≤ *x* < 2.5 , then *y* = *x* – 2 + 2.5 – *x* +

3.6 – *x* = 4.1 – *x*

Again, this will be least if *x* is the highest case y will be

just more than 1.6.

Case **3:** If 2.5 ≤ *x* < 3.6 , then *y* = *x* – 2 + *x* – 2.5 + 3.6

– *x* = *x* – 0.9

This will be least if *x* is least *i.e*. *x* = 2.5 & *y* here will be 1.6

Case **4:** If *x* ≥ 3.6 , then

*y* = *x* – 2 + *x* – 2.5 + *x* – 3.6 = 3*x* – 8.1

The minimum value of this will be at *x* = 3.6 and y = 2.7

Hence, the minimum value of *y* is attained at *x* = 2.5

47. (b) : If *x* – 3 > 0 and *x* + 4 > 0, then using Rule of modulus operation

*x* – 3 + *x* + 4 = 10 *x* = 4.5

If *x* – 3 < 0 and *x* + 4 < 0, then using Rule of modulus operation

–(*x* – 3) – (*x* + 4) = 10 *x* = – 5.5

Now check the given equation by putting the values, *x* = 4.5, we get |4.5 – 3| + |4.5 + 4| = 10 = RHS

*x* = –5.5, we get |–5.5 – 3| + | –5.5 + 4| = 8.5 + 1.5 = 10 = RHS, hence the solutions are

(4.5, –5.5).

[Note: While solving the problems involving modulus, the first step is to remove the modulus sign by using whether *x* = *a* or –*x* = *a* in |*x*| = *a***]**

48. (d) : It is convenient to check with the values taken between the intervals given in the choice, *x* = 1 gives [3*x*] + [4*x*] + [5*x*] = 12 (≠ 14). So, option (a) eliminated.

 gives [3*x*] + [4*x*] + [5*x*] = 14.

So, option (d) is correct. (We do not need to check further)

49. (c) : > 5, *x* ≠ 3,

5| *x* – 3 | < 2

|  |  |
| --- | --- |
| When *x* – 3 > 0  5(*x* – 3) < 2  or, 5*x* < 17   | When *x* – 3 < 0  –5(*x* – 3) < 2  5(*x* – 3) > –2  or 5*x* > 13   |

 is the solution for *x*.

Hence, option (a) is correct.

50. (d) : Case **I:** If *x* ≥ 4

*x* – 2 + *x* – 3 + *x* – 4 ≥ 9

or, 3*x* ≥ 18 or, *x* ≥ 6

Case **II:** If 3 ≤ *x* < 4

*x* – 2 + *x* – 3 – *x* + 4 ≥ 9

*x* ≥ 10 ; not feasible.

 *x* ≤ 0 or *x* ≥ 6.

Case **III:** If 2 ≤ *x* < 3

*x* – 2 – *x* + 3 – *x* + 4 ≥ 9

– *x* ≥ 4

 *x* ≤ 4

Case **IV:** If *x* < 2

– *x* – 2 – *x* + 3 – *x* + 4 ≥ 9

– 3*x* ≥ 0

 *x* ≤ 0

51. (a) : When *x* is not an integer, the floor and ceiling are 1 apart.

In other words:. With this you can express rthe definition of the function.



52. (c) : As we know that



where *n* is any integer. Here *x* =, *n* = 400. So, sum of the

series is 

53. (d) : =

= log*x x* – log*x y* + log*y* *y* – log*y* *x* (log*x* *x* and log*y* *y* = 1)

= 2 – log*x y* – log*y x*

Let, *t* = log*x y*

 

Which can never be positive ; out of given option it can't assume a value of +1. So (d) is answer.

54. (c) : The best way to do this is to take some value and verify.

e.g. 2,  and 1. Thus, *n* = 3 and the sum of the three numbers

= 3.5.

Thus options (a), (b) and (d) get eliminated.

Alternative **method:**

Let the *n* positive numbers be *a*1, *a*2, *a*3 … *an*

*a*1, *a*2, *a*3 … *an* = 1

We know that AM ≥ GM

Hence  (*a*1 + *a*2 + *a*3 + … + *an*) ≥ 

or (*a*1 + *a*2 + *a*3 + … *an*) ≥ *n*

55. (b) : *u* is always negative. Hence, for us to have a minimum value of  , *vz* should be positive. Also for the least value, the numerator has to be the maximum positive value and the denominator has to be the smallest negative value. In other words, *vz* has to be 2 and *u* has to be –0.5.

Hence the minimum value of .

For us to get the maximum value, *vz* has to be the smallest negative value and u has to be the highest negative value. Thus, *vz* has to be –2 and *u* has to be –0.5.

Hence the maximum value of .

56. (c) : *x* + *y* = 1 and *x* > 0, *y* > 0

Taking *x* = *y* =  value of



= = 12.5

It can be easily verified as it is the least value among

options.

57. (d) : *f*(*x*) = max (2*x* + 1, 3 – 4*x*)

So, the two equations are *y* = 2*x* + 1 and *y* = 3 – 4*x*

*y* – 2*x* = 1



Similarly *y* + 4*x* = 3

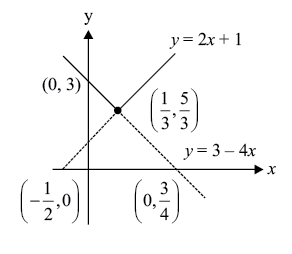


Their point of intersection would be

2*x* + 1 = 3 – 4*x*

6*x* = 2





So, when  then *f*(*x*)max = 3 − 4*x*

And when  then *f*(*x*)max = 2*x* + 1

Hence the min. of this would be at *x* = 

*i.e.* 

Alternative **method:**

As *f*(*x*) = max (2*x* + 1, 3 − 4*x*)

We know that *f*(*x*) would be min at the point of intersection of these curves

*i.e*. 2*x* + 1 = 3 − 4*x*

6*x* = 2

*i.e*. *x* =  Hence min *f*(*x*) is .

58. (d) : When *a* > 0, *b* < 0,

*ax*2 and –*b* | *x* | are non negative for all *x*,

*i.e*. *ax*2 – *b* | *x* | ≥ 0

 *ax*2 – *b* | *x* | is minimum at *x* = 0 when *a* > 0, *b* < 0.

59. (b) : We have

*f*(0) = 03 – 4(0) + *p* = *p*

*f*(1) = 13 – 4(1) + *p* = *p* – 3

If *P* and *P* – 3 are of opp. signs then *p*(*p* – 3) < 0

Hence 0 < *p* < 3.

60. (d) : We can see that *x* + 2 is an increasing function and 5 – *x* is a decreasing function. This system of equation will have smallest value at the point of intersection of the two. *i.e.* 5 – *x* = *x* + 2 or *x* = 1.5. Thus smallest value of *g*(*x*) = 3.5

61. (d) : Using rules of modulus operation, *r* – 6 = 11 or –(*r* – 6) = 11

*r* = 17 or –5

*q* – 6 = 4 or –(*q* – 6) = 4  *q* = 10 or *q* = 2

 is minimum when *q* is minimum and *r* is maximum only if *q* and *r* are both > 0 or < 0.

Here *q* > 0 but *r* < 0, so for minimum value we have to put negative value of *r* along with maximum value of *q*.

Taking *q* = 10 *r* = –5 we get the required minimum value of  = – 2

62. (c) : |5 + 10*x* – 2*x*2|

= 

= 

The maximum value is  – *x* = 0, *i.e*.  which is within the given range [– 1, 3].

63. (c) : Let 

*x*2(*y* – 1) + 2*x*(*y* – 1) + 7*y* – 1 = 0

For x to be real, *D* ≥ 0, 4(*y* – 1)2 – 4(*y* – 1) (7*y* – 1) ≥ 0,

 – 7*y*2(*y* – 1) ≥ 0

 7*y*2 (1 – *y*) ≤ 0, *y* = 0; 1

Hence max *f*(*x*) = 1, min *f*(*x*) = 0.

64. (d) : For maximum profit required number of screws *A* = 30

65. (c) : Maximum profit = Rs. 41

66. (c) : *X* – *X* 2 + 12 ≥ 0 or *X* 2 – *X* – 12 ≤ 0 or

(*X* – 4) (*X* + 3) ≤ 0 *i.e.* –3 ≤ *x* ≤ 4

67. (c) : We think about the "Completion of Squares" method.

*x*2 + 6*x* + 6 = *x*2 + 6*x* + 9 – 3 = (*x* + 3)2 – 3

(*x* + 3)2 – 3 has a range [–3, ∞).

We need to find the range of.

Let us break the range of (*x* + 3)2 – 3 into two ranges [–3, 0] and (0, ∞).

When (*x* + 3)2 – 3 ranges from (0, ∞),  ranges from (0, ∞).

When (*x* + 3)2 – 3 ranges from [–3, 0],  ranges

from.

Overall range = 

68. (b) : Let the last term be *n*, then

*a* + *arn* −1 = 66 .... (1)

and *ar* . *arn* − 2 = 128

 *a*2*rn* − 1 = 128 ..... (2)

From Eqs. (1) and (2),

*a* (66 − *a*) = 128

 *a*2 − 66*a* + 128 = 0

 *a* = 64, 2

69. (b) : Let the first term and common term of the A.P. be *a* and *d* respectively.

Then, (*a* + 5*d*) + (*a* + 14*d*) = (*a* + 6*d*) + (*a* + 9*d*) + (*a* + 11*d*)

 2*a* + 19*d* = 3*a* + 26*d*

 *a* + 7*d* = 0

 8th term is 0.

70. (d) : Let the two numbers be *x* and *y*.

Then, A.M.,



*x* + *y* = 10 ..... (1)

and G.M., 

*xy* = 16

(*x* − *y*)2 = (*x* + *y*)2 − 4*xy*

= 100 − 64 = 36

Or *x* − *y* = ± 6 ..... (2)

Solving Eqs. (1) and (2),

*x* = 8 and *y* = 2 or *x* = 2 & *y* = 8.

71. (d) : The number of terms of the series forms the sum of first *n* natural numbers *i.e.*

.

Thus the first 23 letters will account for the first

=  terms of the series.

The 288th term will be the 24th letter which is *x*.

72. (c) : Assume the number of horizontal layers in the pile be *n*.

So, 

 

 

 

 

 *n* (*n* + 1) (*n* + 2) = 36 × 37 × 38

So, *n* = 36

73. (c) : Let  ..... (1)

  ..... (2)

(1) – (2) gives,

 ..... (3)

 ..... (4)

(3) – (4) gives,

........

 

 

 

 

 



74. (a) : By observing, we see 6 will appear in 5 sets *T*2, *T*3, *T*4, *T*5 and *T*6. Similarly, 12 will also appear in 5 sets and these sets will be distinct from the sets in which 6 appears, *i.e.* *T*8, *T*9, *T*10, *T*11 and *T*12. Thus, each multiple of 6 will appear in 5 distinct sets. Till *T*96, there will be 16 multiplies of 6. These 16 multiples of 6 will appear in 16 × 5 = 80 sets.

75. (b) : Arithmetic mean is more by 1.8 means sum is more by 18.

So *ba* – *ab* = 18

*b* > *a* because sum has gone up, e.g. 31 – 13 = 18

Hence, *b* – *a* = 2

76. (c) : Let the 6th and the 7th terms be *x* and *y*.

Then 8th term = *x* + *y*

Also *y*2 – *x*2 = 517

 (*y* + *x*)(*y* – *x*) = 517 = 47 × 11

So *y* + *x* = 47

*y* – *x* = 11

Taking *y* = 29 and *x* = 18, we have 8th term = 47,

9th term = 47 + 29 = 76 and 10th term = 76 + 47 = 123.

77. (c) : *t*1 = 20 *tn* = 0





78. (d) : 





Ratio of 11th terms

= 

= 

79. (c) : 



Hence, option (c) is correct.

80. (c) : *a*1 = 1, *an*+1 – 3*an* + 2 = 4*n*

*an*+1 = 3*an* + 4*n* – 2

when *n* = 2 then *a*2 = 3 + 4 – 2 = 5

when *n* = 3 then *a*3 = 3 × 5 + 4 × 2 – 2 = 21

from the options, we get an idea that *an* can be expressed in a combination of some power of 3 & some multiple of 100.

(a) 399 – 200; tells us that *an* could be: 3*n*–1 – 2 × *n*;

but it does not fit *a*1 or *a*2 or *a*3

(b) 399 + 200; tells us that *an* could be: 3*n*–1 + 2 × *n*;

again, not valid for *a*1, *a*2 etc.

(c) 3100 – 200; tells 3*n* – 2*n*: valid for all *a*1, *a*2, *a*3.

(d) 3100 + 200; tells 3*n* + 2*n*: again not valid.

so, (c) is the correct answer.

81. (a) : Let us say there are only 3 questions.

Thus, there are 23 – 1 = 4 students who have done 1 or more questions wrongly, 23 – 2 = 2 students who have done 2 or more questions wrongly and 23 – 3 = 1 student who must have done all 3 wrongly.

Thus total number of wrong answers = 4 + 2 + 1 = 7 = 23 – 1 = 2*n* – 1.

In our question, the total number of wrong answers

= 4095 = 212 – 1. Thus *n* = 12.

82. (c) : *f*1 *f*2 = *f*1(*x*) *f*1(–*x*)



= 

*f*1*f*1(–x) = 0*x*

Similarly *f*2*f*3 = –(*f*1(–*x*))2 ≠ 0 for some *x*

*f*2*f*4 = *f*1(–*x*). *f*3(–*x*)

= –*f*1(–*x*) *f*2(–*x*)

= –*f*1(–*x*) *f*1(*x*) = 0*x*

83. (b) : Check with options Option (b)

*f*3(–*x*) = –*f*2(–*x*)

= –*f*1(*x*)

 *f*1(*x*) = –*f*3(–*x*) *x*

84. (b) : 

= 

= 

= 

= 

= 

85. (a) : Assume some values of *A* and *B* and substitute in the options to get the answer.

Let us say *A* = 2, *B* = 4.

Then (*A*, *B*) = 

and (3, 2) = 3 × 2 = 6 *i.e*. the sum of *A + B* = 2 + 4 = 6

Hence, option (a) is the answer.

86. (d) : Again, this can be solved by assuming some values of *A, B* and *C*.

87. (b) : It is not linear in *x* and *y*, that's why option (a) is neglected. It also can't be exponential. By substituting *X* and *Y* in *y* = *a* + *bx* + *cx*2 we see that it gets satisfied.

88. (d) : *f*(*x* + 1, *y*) = *f*[*x*, *f*(*x*, *y*)]

put *x* = 0, *f*(1, *y*) = *f*[0, *f*(0, *y*)] = *f*[0, *y* + 1]

= *y* + 1 + 1 = *y* + 2

put *y* = 2, *f*(1, 2) = 4.

89. (b) : As graph is symmetrical about *y* - axis, we can say function is even, so *f*(*x*) = *f*(–*x*).

90. (d) : We see from the graph. Value of *f*(*x*) in the left region is twice the value of *f*(*x*) in the right region.

So, 2*f*(*x*) = *f*(–*x*) or 6*f*(*x*) = 3*f*(–*x*).

91. (c) : *f*(–*x*) is replication of *f*(*x*) about *y* axis, –*f*(*x*) is replication of *f*(*x*) about *x* - axis and –*f*(–*x*) is replication of *f*(*x*) about *y* - axis followed by replication about *x* - axis. Thus given graph is of *f*(*x*) = –*f*(–*x*).

92. (c) : Putting the actual value in the functions, we get the required answers.

*m*(*a*, *b*, *c*) = –5, *M*(*a*, *b*, *c*) = 2

So  is maximum *i.e*.

=

93. (c) : *m*(*a*, *b*, *c*) = min (*a* + *b*, *c*, *a*);

–*M*(–*a*, *a*, –*b*) = –max (0, –*b*, –*a*);

*m*(*a* + *b*, *b*, *c*) = min (*a* + 2*b*, *c*, *a* + *b*)

Since, *a, b* & *c* are all negative, we will get minimum value in third case.

94. (c) : *m*(*M*(*a* – *b*, *b*, *c*), *m*(*a* + *b*, *c*, *b*), – *M*(*a*, *b*, *c*))

= *m*(3, 4, –6) = –6.

95. (d) : 







96. (a) : When *x* is negative, *f*(*x*) = 1 + *x*

*f*(–1) = 1 – 1 = 0;

*f* 2(–1) = *f*[*f*(–1)] = *f*(0) = 1;

*f* 3(–1) = *f*[*f* 2(–1)] = *f*(1) = 

*f* 4(–1) = *f*[*f* 3(–1)] =

97. (b) : *f*(*x*).*f*(*y*) = *f*(*xy*)

Given, *f*(2) = 4

We can also write;

*f*(2) = *f*(2 × 1) = *f*(2) × *f*(1)

OR *f*(1) × 4 = 4

 *f*(1) = 1

Now we can also write,







Hence, option (b) is the correct choice.

98. (b) : Let *h*(*x*) = *f*(*x*) + *g*(*x*) where *f* and *g* are odd

 *h*(– *x*) = *f*(– *x*) + *g*(– *x*) = – *f*(*x*) – *g*(*x*) = – (*f* (*x*) + *g*(*x*)

= – *h*(*x*), so, *h* is also odd.

Hence, option (b) is correct.

99. (b) : 

= 

= 

= 

100. (b) : Since = 

(important relation)

Given that *f*(10) = 1001  10*n* + 1 = 1001  *n* = 3

*f*(20) = (20)3 + 1 = 8001

Hence option (b) is correct.

101. (a) : *f*(0) = 0 + 0 + *c* = 3  *c* = 3

*f*(2) = 8 + 2*b* + 3 = 1  *b* = − 5

 *f*(1) = 2 − 5 + 3

= 0

102. (c) : We have *f*(*x* + *f*(*x*)) = 5 *f*(*x*)

Putting *x* = 1

*f*(1 + *f*(1)) = 5 *f*(1)

 *f*(1 + 4) = 5 × 4

 *f*(5) = 20

Now putting *x* = 5

 *f*(5 + *f*(5)) = 5 *f*(5)

 *f*(5 + 20) = 5 × 20

 *f*(25) = 100

103. (d) : 







Hence, option (d) is correct.

104. (c) : *f*(*x*) = 1 if *x* < 1

= 2 if *x* = 1

= *x* if *x* > 1

*f*(*f*(*f*(*f*(*f*(1))))) = *f*(*f*(*f*(*f*(2)))) = *f*(*f*(*f*(2))) = *f*(*f*(2)) = *f*(2) = 2.

Hence, option (c) is correct.

105. (c) : 

Hence, option (c) is correct.

106. (a) : The graph is symmetric about *y* axis.

So, *f*(–*x*) = *f*(*x*).

Hence, option (a) is correct.

107. (b) : The graph is symmetric about origin *f*(–*x*) = –*f*(*x*).

Hence, option (b) is correct.

108. (d) : In the graph, for some point '*x*', ther are two corresponding '*y*' which violates the definition of function.

Hence, option (d) is correct.

109. (a) : The graph is symmetric about *y* axis.

So *f*(–*x*) = *f*(*x*).

Hence, option (a) is correct.

110. (d) : In the graph, for some point '*x*', there are two corresponding '*y*' which violates the definition of function.

Hence, option (d) is correct.

111. (b) : The graph is symmetric about origin *f*(–*x*) = –*f*(*x*).

Hence, the option (b) is correct.

112. (d) : (*q* + 1) $ *q* = (*q* + 1)*q*

Ұ *q* = *q* + 1

\* *q* = *q*2 – 1

Ұ(\* *q*) = *q*2 – 1 + 1 = *q*2

For *q* is an integer greater than 1,

(*q* + 1)*q* is the greatest.

113. (c) : Putting *n* = 1, we get *f*(2) + *f*(0) = 2*f*(1)  *f*(2) = 2*f*(1)

putting *n* = 2, we get *f*(3) + *f*(1) = 2*f*(2)  *f*(3) = 2(2*f*(1)) – *f*(1)  *f*(3) = 3*f*(1)

 *f*(*n*) = *nf*(1) for *n* = 1, 2, 3, ..... *n*.

Cross - check: Given *f*(*n* + 1) + *f*(*n* – 1) = 2*f*(*n*)

 *f*(*n* + 1) + (*n* – 1) *f*(1) = 2*nf*(1)

 *f*(*n* + 1) = (2*n* – *n* + 1) *f*(1) = (*n* + 1) *f*(1)

Thus, *f*(*n*) = *nf*(1), *n**N*.

114. (c) : |*x* + *y*| + |*x* – *y*| = 4

Replace “+*x*” by “– *x*” & “+*y*” by “–*y*” everywhere in the

curve: we again get the same equation.

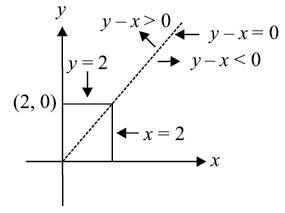
 curve is symmetric in the 4-quadrants of *X*–*Y* plane.

In I-quadrant (*x*, *y* > 0)

|*x* + *y*| + |*x* – *y*| = 4

= 

Or 



We can now plot graph:

Area in I-quadrant = (2)2 = 4 unit2

total area of |*x* + *y*| + |*x* – *y*| = 4 is:

4 × (area of I-quadrant) = 4 × 4 = 16 sq. unit.

115. (c) : 2*x* – *x* – 1 = 0

 2*x* – 1 = *x*

If we put *x* = 0, then this is satisfied and if we put *x* = 1, then also this is satisfied. Now we put *x* = 2, then this is not valid.

Hence, we have two values.

116. (d) : Here, *F*(2) = –1, *F*(–2) = 1, *G*(–2) = 2. On checking the options we get,

*F*(2) – *G*(–2) = 4

– 1 – 2 ≠ 4

*F*(–2) – *G*(2) = 4

2 – 1 ≠ 4

F(2) + G(–2) = – 4

– 1 + 2 ≠ – 4

Hence, option (d) is the answer.

117. (b) : Here, *F*(2) = –1, *F*(–2) = 2, *G*(2) = –2, *G*(–2) = 3. On checking the options we get,

*F*(2) – *G*(–2) = 4

–1 –3 ≠ 4

*F*(–2) – *G*(2) = 4, true

2 + 2 = 4

*F*(2) + *G*(–2) = –4

–1 + 3 ≠ –4

Hence, option (b) is the answer.

118. (d) : The function of the given graph can be written as

*y* = 0 for 0 < *x* ≤ 

*y* = 1 for  < *x* ≤ 1

*y* = 2 for 1 < *x* ≤  and so on.

or *y* = 0 for 0 < 2*x* ≤ 1

*y* = 1 for 1 < 2*x* ≤ 2

*y* = 2 for 2 < 2*x* ≤ 3 ⇒ *y* = [2*x*]

119. (c) : *f*(*x*) = *x*2 + 6*x* + 5

or *f*(*x*) = *x*2 + 6*x* + 9 – 4 or *f*(*x*) = (*x* + 3)2 – 4

*f*(*x*) is minimum when *x* = –3.

120. (c) : *f*(*x*) = (*x* + 3)2 – 4

*f*(–3) = –4 is minimum value of function.

**Solution Exercise – Difficult**

1. (b) : Given 

 *x* = *a* (2*x* + *y* + *z*), *y* = *a* (*x* + 2*y* + *z*), *z* = *a* (*x* + *y* + 2*z*)

 *x* + *y* + *z* = *a* (4*x* + 4*y* + 4*z*)

 4*a* = 1  *a* = 

2. (b) : *a* + *b* + *c* = 0

 *a*2 + (*b* + *c*)2 or *a* = −*b* −*c*

 Given expression



= 

= 

= 

3. (a) : (*a* + *b* + 2*c* + 3*d*) (*a* − *b* − 2*c* + 3*d*)

= (*a* − *b* + 2*c* − 3*d*) × (*a* + *b* − 2*c* − 3*d*)

 [(*a* + *b*) + (2*c* + 3*d*)] [(*a* − *b*) − (2*c* − 3*d*)]

= [(*a* − *b*) + (2*c* − 3*d*)] [(*a* + *b*) − (2*c* + 3*d*)]

 (*a* + *b*) (*a* − *b*) − (*a* + *b*) (2*c* − 3*d*) + (*a* − *b*) (2*c* + 3*d*) − (2*c* + 3*d*) (2*c* − 3*d*)

= (*a* − *b*) (*a* + *b*) − (*a* − *b*) (2*c* + 3*d*) + (*a* + *b*) (2*c* − 3*d*) − (2*c* − 3*d*) (2*c* + 3*d*)

 2 (*a* − *b*) (2*c* + 3*d*) = 2 (*a* + *b*) (2*c* − 3*d*)

 2*ac* + 3*ad* − 2*bc* − 3*bd* = 2*ac* − 3*ad* + 2*bc* − 3*bd*

 6*ad* = 4*bc*

2*bc* = 3*ad*

4. (d) : Equation (ii) can be written as

40.3*x* × 90.2*y* = 8 ×

(22)0.3*x* (32)0.2*y* = 8.

20.6*x* 30.4*y* = 23.  = 23 . 

0.6*x* = 3  x = 5

and 0.4*y* = 

 *y* = 2

5. (b) : 2*x* + *y* = 40

*x* ≤ *y*

 *y* = 40 – 2*x*

Values of *x* and *y* that satisfy the equation

|  |  |
| --- | --- |
| ***x*** | ***y*** |
| 1 | 38 |
| 2 | 36 |
| 3 | 34 |
| . | . |
| . | . |
| . | . |
| 13 | 14 |
|  |  |

 13 values of (*x*, *y*) satisfy the equation such that *x* ≤ *y*

6. (d) : log*y x* = *a* log*z y* = *b* log*x z* = *a* × *b*





= 

=  = (log*y x*)3 = (*ab*)3

So, *ab* − *a*3*b*3 = 0

Or, *a* × *b* (1 − *a*2*b*2) = 0

 *ab* = ±1

Only option (d) does not satisfy. Hence (d).

7. (c) : Given equation is *x* + *y* = *xy*

 *xy* – *x* – *y* + 1 = 1

 (*x* – 1)(*y* – 1) = 1

*x* – 1= 1 & *y* −1= 1or *x* −1= –1 & *y* – 1= –1

Clearly (0, 0) and (2, 2) are the only pairs that will satisfy the equation.

8. (c) : If 

then there are only two possibilities.

(i)

If *a* + *b* + *c* ≠ 0, then



= 

(ii)

If *a*+ *b* + *c* = 0, then

*b* + *c* = – *a*

*c* + *a* = – *b*

*a* + *b* = – *c*

Hence 

Similarly, 

Therefore option (c) is the correct one  or –1

9. (b) : *u* = (log2*x*)2 – 6log2*x* + 12

*xu* = 256

Let log2*x* = *y*  *x* = 2*y*

*xu* = 28  *uy* = 8  *u* = 

 = *y*2 − 6*y* + 12  *y*3 − 6*y*2 + 12*y* − 8 = 0

 (*y* − 2)3 = 0  *y* = 2

*x* = 4, *u* = 4

10. (a) : It is given that *p* + *q* + *r* ≠ 0 , if we consider the first option, and multiply the first equation by 5, second by –2 and third by –1, we see that the coefficients of *x*, *y* and *z* all add up-to zero.

Thus, 5*p* – 2*q* – *r* = 0 No other option satisfies this.

11. (c) : If *p* = *q* = *r* = 1, then expression = 1

Check the choice only, one choice gives the value of expression = 1.

12. (d) : *ax* + *by* + *cz* = 0

*a*2*x* + *b*2*y* + *c*2*z* = 0



 *x* = *k bc*(*c* – *b*), *y* = *k ac*(*a* – *c*), *z* = *k ab*(*b* – *a*)

dividing by *abc*



13. (d) : 

= 

= 

14. (a) : *t* − 2 = 

(*t* − 2)3 = 4 + 2 + 

 *t*3 − 23 − 3 × 2 × *t* (*t* − 2) = 6 + 6*t* − 12

 *t*3 − 6*t*2 − 2 = 0

15. (d) : 

The value of *x* is greater than 2, let us try it for *x* = 2 and 3

For *x* = 2, *x*3 − 6*x*2 + 6*x* = −4

For *x* = 3, *x*3 − 6*x*2 + 6*x* = −9

 *x*3 − 6*x*2 + 6*x* < 0 for *x* = 2 and 3

Hence, option (d) is our answer.

16. (b) : Let *f*(*x*) = *ax*2 + *bx* + *c*

At *x* = 1, *f*(1) = *a* + *b* + *c* = 3

At *x* = 0, *f*(0) = *c* = 1

The maximum of the function *f*(*x*) is attained at



*a* = –2 and *b* = 4

Therefore *f*(*x*) = –2*x*2 + 4*x* + 1

Therefore *f*(10) = –159

17. (b) : (*a* + *b* + *c* + *d*)2 = (4*m* + 1)2

Thus, *a*2 + *b*2 + *c*2 + *d*2 + 2(*ab* + *ac* + *ad* + *bc* + *bd* + *cd*)

= 16*m*2 + 8*m* + 1

*a*2 + *b*2 + *c*2 + *d*2 will have the minimum value if (*ab* + *ac*

+ *ad* + *bc* + *bd* + *cd*) is the maximum.

This is possible if *a* = *b* = *c* = *d* = (*m* + 0.25) … since *a*

+ *b* + *c* + *d* = 4*m* + 1

In that case 2,

((*ab* + *ac* + *ad* + *bc* + *bd* + *cd*)

= 12(*m* + 0.25)2 = 12*m*2 + 6*m* + 0.75

Thus, the minimum value of *a*2 + *b*2 + *c*2 + *d*2

= (16*m*2 + 8*m* + 1) – 2(*ab* + *ac* + *ad* + *bc* + *bd* + *cd*)

= (16*m*2 + 8*m* + 1) – (12*m*2 + 6*m* + 0.75)

= 4*m*2 + 2*m* + 0.25

Since it is an integer, the actual minimum value

= 4*m*2 + 2*m* + 1

18. (a) : Let α is the common root.

 α3 + 3α2 + 4α + 5 = 0

α3 + 2α3 + 7α + 3 = 0

α2 − 3α + 2 = 0

α = 2, α = 1

But the above values of α do not satisfy any of the equations. Thus, no root is common.

19. (d) :   *A*2(*x* – 1) + *B*2*x* = *x*2 – *x*

This is a quadratic equation.

Hence, number of roots = 2 or 1 (1 in the case when both roots are equal).

20. (a) : Roots are real and unequal  *D* > 0

 (4*k* – 1)2 > 4 (2*k*2 – 1) ×2  *k* < 

Roots are negative, ∞ + β < 0  



 

Hence option (a) is correct.

21. (d) : Do not solve to find the common root, put the options in the given equations to check which satisfies.

(*a*) *a* = 0, *a* = . One quadratic equation becomes linear.

 *a* ≠ 0, *a* ≠ 

So, *a* =  is the only option available. Further checking is not needed.

Hence. option (d) is correct.

22. (a) : Let α be the common root.

 α2 + *a*α + *b* = 0 ..... (1)

α2 + *b*α + *a* = 0 ..... (2)

Substracting (2) from (1)

(*a – b*)α + (*b – a*) = 0 or (*a – b*)α = (*a – b*)

or, α = 1. Putting α = 1 in (1)

we have *a + b* = – 1.

Hence, option (a) is correct.

23. (b) : 6*x*3 – *ax*3 + 6*x* – 1 = 0 has three roots α, β, γ and their reciprocals are in A.P.

 

[If α, β, γ are roots of *ax*3 + *bx2* + *cx* + *d* = 0, then α + β + γ =

, αβ + βγ + γα =  and αβγ =]

α + β + γ = 0 ..... (1)

αβ + βγ + γα =  ..... (2)

αβγ =  ..... (3)

After dividing (2) by (3), we get 



By putting this value in equation (2) and (3)



 

 *a* = 22

24. (b) : *x*2 + *a* |*x*| + 1 = 0

*x* > 0, *x*2 + *ax* + 1 = 0



If *a*2 – 4 > 0, then all the roots to this conditions are real and positive. So from this condition, we will get 2 roots

*x* < 0, *x*2 – *ax* + 1 = 0.

 

As a is negative, both roots will be negative.

So, we will get 4 roots or no real root in this equation depending upon value of *a*.

25. (a) : Since *a, b, c* and *d* are roots of

3*x*4 + 2*x*3 + 7*x*2 + *x* + 2 = 0, we have,

3(*x* – *a*) (*x* – *b*) (*x* – *c*) (*x* – *d*) = 3*x*4 + 2*x*3 + 7*x*2 + *x* + 2

Putting *x* = 1 we have

(1 – *a*) (1 – *b*) (1 – *c*) (1 – *b*) = 

26. (c) : *f*(*x*) = *x*2 – 5*x* + 6 < 0. So, (*x* – 3) (*x* – 2) < 0.

Hence, 2 < *x* < 3. *g*(*y*) = *y*2 + 2*y* + 1 is a perfect square.

For *f*(*x*) . *g*(*y*) < 0, *x* must lie between 2 and 3 and *y* can take any real value except '– 1'.

27. (c) : The given value is maximum when the decimal number is just short of whole number,

*i.e*. say *p = q = r* = 0.99

The given function = [0.99 + 0.99 + 0.99] – {[0.99] + [0.99] + [0.99]} = [2.7] – 0 = 2.

28. (c) : *f*1(*x*) = |*x* – 1| – 1

for *x* > 0, *f*1(*x*) = *x* – 2,

for *x* < 0, *f*1(*x*) = –*x*, also *f*1(1) = –1.

*f*1(*x*) is represented by *G*2

*f*2(*x*) = ||*x* – 1| – 1|

for *x* > 0, *f*2(*x*) = |*x* – 2|

for *x* < 0, *f*2(*x*) = *x*

for *x* = 0, *f*2(*x*) = 0

*f*2(*x*) is represented by *G*3

It can be proved that *f*3(*x*) is represented by *G*1 and *f*4(*x*) is, represented by *G*4

29. (a) : The given equality is 960 ≤ |*x* – 8| + |*x* – 88| + |*x* – 888| ≤ 1000.

Condition **I:** If *x* ≤ 8, then

960 ≤ – (*x* – 8) – (*x* – 88) – (*x* – 888) ≤ 1000

960 ≤ – *x* + 8 – *x* + 88 – *x* + 888) ≤ 1000

960 ≤ –3*x* + 984 ≤ 1000  3*x* ≤ 24 or 3*x* ≥ –16

–5.33 ≤ *x* ≤ 8.

Condition **II:** If 8 ≤ *x* ≤ 88, then

960 ≤ *x* – 8 – *x* + 88 – *x* + 888 ≤ 1000

 960 ≤ –*x* + 968 ≤ 1000

*x* ≥ 8 or *x* ≥ –32

–32 ≤ *x* ≤ 8. (No value of *x* is possible for this region)

Condition **III:** If 88 ≤ *x* ≤ 888, then

960 ≤ *x* – 8 + *x* – 88 – *x* + 888 ≤ 1000

960 ≤ *x* + 792 ≤ 1000

*x* ≥ 168 and *x* ≤ 208.

168 ≤ *x* ≤ 208.

Condition **IV:**  If *x* ≥ 888, then

960 ≤ *x* – 8 + *x* – 88 + *x* – 888 ≤ 1000

960 ≤ 3*x* – 984 ≤ 1000

3*x* ≥ 1944 and 3*x* ≤ 1984

*x* ≥ 648 and *x* ≤ 661.33

648 ≤ *x* ≤ 661.33 (No value of *x* is possible for this region)

So, the required values of *x* are 168 ≤ *x* ≤ 208.

30. (b) : *xyz* = 4

*y* – *x* = *z* – *y*

2*y* = *x* + *z*

yis the AM of *x*, *y*, *z*.

Also 

AM ≥ GM



Therefore, the minimum value of *y* is.

31. (d) : Let *f*(*x*) = (*x*2 – 6*x* + 8) (*x*2 – 14*x* + 48) + 20

= (*x* – 2) (*x* – 4) (*x* – 6) (*x* – 8) + 20

= (*x* – 2) (*x* – 8) (*x* – 4) (*x* – 6) + 20

= (*x*2 – 10*x* + 16) (*x*2 – 10*x* + 24) + 20

Put *x*2 – 10*x* = *y* = (*y* + 16) (*y* + 24) + 20

= *y*2 + 40*y* + 384 + 20

= (*y* + 20)2 + 4

Minimum value of given function = 4. (At *x*2 – 10*x* + 20 = 0.)

32. (d) : We know, (*a* + *b* + *c*)2 = *a*2 + *b*2 + *c*2 + 2(*ab* + *ca* + *bc*) ≥ 0

or, 1 + 2(*ab* + *ca* + *bc*) ≥ 0  *ab* + *ca* + *bc* ≥ 

Also, we know *a*2 + *b*2 + *c*2 ≥ *ab* + *bc* + *ca* or,

*ab* + *bc* + *ca* ≤ 1 [ *a*2 + *b*2 + *c*2 = 1]

 Required interval of *bc* + *ca* + *ab* is.

So, option (d) is correct.

33. (a) :  and it is given that *xy* + *yz* + *zx* < 0





The term 

[as (*x* + *y* + *z*)2 ≥ 0 and *xy* + *yz* + *zx* < 0]

 *u* ≤ –2

34. (d) : If *p*, *q*, *r*, *s* are in H.P.

  are in A.P.

 

 

35. (a) : log*x a*,  and log*b x* are in G.P., then

= (log*x a*) × (log*b x*)

 *ax* = log*b a*

 *x* log *a* = log*a* (log*b a*)

 *x* = log*a* (log*b a*)

36. (d) : The number of members in the set *S* = *nC*2, where *n* is

greater than = 4

Each member of *S* has two distinct numbers.

Let us say (1, 2) is one of the members of *S*.

 To find the number of enemies each member of *S* will have be, we will exclude 1 and 2 from '*n*' and find the total number of possibilities *i.e*. 

Alternative **Method for questions 37 and 38:**

For *n* = 6, the number of elements in the set *S* = {(1, 2), (1, 3),

(1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6) and (5, 6)

Lets consider the member (1, 2).

Number of enemies for this member is 6, *i.e*. (3, 4), (3, 5),

(3, 6), (4, 5) (4, 6) and (5, 6).

Checking by options, this is only satisfied by 

Hence  is the correct choice.

37. (d) : Considering any two members of *S*, that are friends there will be 1 number of the pairs that will be common. The common element of these pairs will have *n* – 3 pairs, with the remaining *n* – 3 elements. There will be one more member made up of the remaining two constituent elements which are not same. In total there are *n* – 3 + 1 = n – 2 other members of *S* that are common friends of the chosen two pairs or numbers.

Alternative **Method for questions 37 and 38:**

For *n* = 6, the number of elements in the set *S* = {(1, 2), (1, 3),

(1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5),

(3, 6), (4, 5), (4, 6) and (5, 6)

Lets consider the member (1, 2).

For *n* = 6 lets consider the members (1, 2) and (1, 3)

Friends of the member (1, 2) in the set *S* are (1, 4), (1, 5),

(1, 6), (2, 3), (2, 4), (2, 5), (2, 6).

Friends of the member (1, 3) in the set *S* (1, 4), (1, 5),

(1, 6), (2, 3), (3, 4), (3, 5), (3, 6).

The number of members of *S* that are common friends to the above member are 4, *i.e.* (1, 4), (1, 5), (1, 6), (2, 3).

So the answer is *n* – 2.

38. (d) : Let number of elements in progression be *n*, then

1000 = 1+ (*n* − 1)*d*

(*n* − 1)*d* = 999 = 33 × 37

Possible values = 3, 37, 9, 111, 27, 333, 999

Hence, 7 progressions are possible.

39. (c) : Using log *a* – log *b* =,

= (where *y* = 2*x*)

Solving we get *y* = 4 or 8 *i.e*. *x* = 2 or 3. It cannot be 2 as log of negative number is not defined.

40. (d) : Sum of  *n* terms.

Such problem must be solved by taking the value of number

of terms.

Let’s say 2 and check the given option. If we look at the sum of 2 terms of the given series it comes out to be 

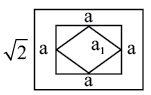
Now look at the option and put number of terms as 2,

only option (d) validates the above mentioned answer.

As, 

41. (c) : *P*2 = 4 × *a*1 = 4 × *a*

Also, *A*2 = 





= 

= 

42. (a) : Coefficient of 

*S* = 2 + 5*x* + 9*x*2 + 14*x*3 + ...

*xS* = 2*x* + 5*x*2 + ....

*S*(1 – *x*) = 2 + 3*x* + 4*x*2 + 5*x*3 + ...

Let *S*1 = *S* (1 – *x*) *S*1 = 2 + 3*x* + 4*x*2 + ...

*S*1 (1 – *x*) = 2 + *x* + *x*2 + ....

*S*1 (1 – *x*) = 2 + 

*S* (1 – *x*)2 = 2 +  

***Solutions for questions 43 and 44:***

Using the given expressions —

*a*1 = *p* *b*1 = *q*

*a*2 = *pq* *b*2 = *q*2

*a*3 = *p*2*q* *b*3 = *pq*2

*a*4 = *p*2*q*2 *b*4 = *pq*3

*a*5 = *p*3*q*2 *b*5 = *p*2*q*3

*a*6 = *p*3*q*3 *b*6 = *p*2*q*4

and so on

43. (a) : *an* + *bn* (*n* is even) = 

44. (d) : *an* + *bn* (*n* is odd)

= 

Substituting



Substituting *n* = 7, *an* + *bn* > 0.01

Substituting *n* = 9, *an* + *bn* < 0.01

Hence smallest value of *n* is 9

45. (d) : *x*0 *= x*

*x*1 *= –x*

*x*2 *= –x*

*x*3 *= x*

*x*4 *= x*

*x*5 *= –x*

*x*6 *= –x*

………..

Choices (a), (b), (c) are incorrect.

46. (a) : *f*(1) + *f*(2) + *f*(3) + …. + *f*(*n*) = *n*2*f*(*n*) , *f*(1) = 3600.

For *n* = 2:

 *f*(1) + *f*(2) = 22 *f*(2)  *f*(2) = 

For *n* = 3:

 

 

Similarly



Therefore, *f*(9) = 80

47. (d) : *g*(*x* + 1) + *g*(*x* –1) = *g*(*x*)

*g*(*x*+ 2) + *g*(*x*) = *g*(*x* + 1)

Adding these two equations we get

*g*(*x* + 2) + *g*(*x* – 1) = 0

 *g*(*x* + 3) + *g*(*x*) = 0

 *g*(*x* + 4) + *g*(*x* + 1) = 0

 *g*(*x* + 5) + *g*(*x* + 2) = 0

 *g*(*x* + 6) + *g*(*x* + 3) = 0  *g*(*x* + 6) – *g*(*x*) = 0

48. (a) : *g*2 = *g* \* *g* = *h*

*g*3 = *g*2 \* *g* = *h* \* *g* = *f*

*g*4 = *g*3 \* *g* = *f* \* *g* = *e*

 *n* = 4

49. (d) : *f*  [*f* \* {*f*  (*f* \* *f* )}]

= *f*  [*f* \* {*f**h*}]]

= *f*  [*f* \* *e*}]

= *f*  [*f* ]]

= *h*

50. (a) : *e*8 = *e*2 \* *e*2 \* *e*2

= *e* \* *e* \* *e*

= *e*

If we observe *a* \* (anything) = *a*

 *a*10 = *a*

 {*a*10 \* (*f*10*g*9 )}*e*8

= *a**e*

= *e*

51. (d) : *x* – 1 ≤ [*x*] ≤ *x*

2*x* + 2*y* – 3 ≤ *L*(*x*, *y*) ≤ 2*x* + 2*y*  *a* – 3 ≤ *L* ≤ *a*

2*x* + 2*y* – 2 ≤ *R*(*x*, *y*) ≤ 2*x* + 2*y*  *a*– 2 ≤ *R* ≤ *a*

Therefore, *L* ≤ *R*

[Note: Choice (b) is wrong, otherwise choice (a) and choice (c) are also not correct. Choose the numbers to check.**]**

52. (d) : 

when *x* and *y* are positive

thus for *x* + *y* > 1, (*x* + *y*)0.5 < (*x* + *y*)2,

therefore *f*(*x*, *y*) < *g*(*x*, *y*)

we can therefore eliminate answer option a if x and y are both negative then *f*(*x*, *y*) = (*x* + *y*)2 and *g*(*x*, *y*) = – (*x* + *y*).

Now for –1 < *x* + *y* < 0, (*x* + *y*)2 < –1*x* + *y*,

Therefore *f*(*x*, *y*) < *g*(*x*, *y*)

Thus, answer option b is eliminated. As in evident from the above discussion, for *x* and *y* > –1, we cannot again guarantee that *f*(*x*, *y*) > *g*(*x*, *y*).

53. (c) : When 0 ≤ *x, y* < 0.5, *x + y* may be < 1 or 1, so given statement (a) can be true or false.

When *x, y* < –1, again statement (b) can be true or false.

When *x, y* > 1, *x + y* > 1 hence *f*(*x, y*) < *g*(*x, y*).

*f*(*x, y*) > *g*(*x, y*)

Thus, statement (c) given in necessarily false.

54. (b) : When *x* + *y* = 1 we have (*x* + *y*)2 = (*x* + *y*)0.5

*i.e.* *f*(*x*, *y*) = *g*(*x*, *y*).

Thus answer is (b).

55. (c) : Since *f*(0) = *p* is given, so, *f*(0) needs to be evaluated first. In the given function put *x* = *y* = 0  *f*(0) + *f*(0) = 2*f*(0) . *f*(0)  2*f*(0) – 2*f*(0) *f*(0) = 0  *f*(0) = 1 or 0

Now put *x* = 0 in the function,  *f*(*y*) + *f*(–*y*) = 2*f*(0) *f*(*y*)

When *f*(0) = 0 = *p*, then *f*(*y*) + *f*(–*y*) = 0  *f* is odd.

f(0) = 1 = *p*, then *f*(*y*) + *f*(–*y*) = 2*f*(*y*)  *f* is even.

Hence, option (c) is correct.

56. (b) : Evaluation involves a number of composite functions *f0 g* and *g0 f*.

Hence at first we find and 

.

Since *f0 g*(*x*) = *g0 f*(*x*) = *x*, so these functions if act repeatedly then it will still give the value of *x*.

Hence their product = *x*2.

Hence option (b) is correct.

57. (d) : 

= 

Similarly, 

Now, 

 *g*(*x* + 1) [2*x* + 3] = (*x* + 2) [*g*(*x* + 1) + 1]

 

58. (a) : Given *k*(*f*(*x*)) = 3*x*2 + 7*x*, so

*k*(*f*(*k*(*x*))) = 3{*k*(*x*)2} + 7*k*(*x*) ..... (1)

*f*(*k*(*x*)) = *x*2 + 5*x* – 5

 *k*(*f*(*k*(*x*))) = *k*(*x*2 + 5*x* – 5) ..... (2)

From (i) and (ii) *k*(*x*2 + 5*x* – 5) = 3{*k*(*x*)}2 + 7*k*(*x*)

To find *k*(–5), if we put *x*2 + 5*x* – 5 = –5, then *x* = 0, –5 put *x* = –5

*k*(–5) = 3*k*(–5)2 + 7 *k*(–5)

*k*(–5) = –2, 0

–2 could be the value of *k*(–5).

Hence, option (a) is correct.

59. (d) : 





When, there will be 121 such pairs equal to 1.

Middle term =  (can be obtained by putting *x* =  in *f*(*x*).

Hence required sum = 121 + 0.5 = 121.5

60. (d) : From the given data,

β(*x* – 1) = γ(*x* – 2) β(*x*) = α(*x*)

γ(*x* + 1) = α(*x* + 2) and γ(*x* – 2) = α(*x* – 1)

Now the whole equation can be converted in terms of α,

α(*x*) = β(*x* – 1) + γ(*x* + 1) α(*x*) = γ(*x* – 2) + γ(*x* + 1)

α(*x*) = α(*x* – 1) + α(*x* + 2) ..... (1)

and β(*x* – 1) = α(*x* – 1)

Substituting the values of x in equation (i), we get α(0) = 0, α(1) = 1, α(2) = 2, α(3) = 1, α(4) = 1, α(5) = –1, α(6) = 0, α(7) = –2, α(8) = 1, α(9) = –2. As '*x*' goes up, the absolute value of α(*x*) does not show any trend.

61. (d) : "(*x*, *y*) ← (*z*, *w*)"

I. *x* < *z* or

II. *x* = *z* and *y* > *w*

"(*z*, *w*) ← (*r*, *s*)"

III. *x* < *r* or

IV. *z* = *r* and *w* > *s*

one of I and II and one of III and IV must be true.

Case **I:** I is true, then anyone of III or IV is true, it follows that *x* < *r*.

Case **II:** If II is true and III is true, then also *x* < *r*.

Case **III:** If II and IV is true then, it follows that *x* = *z* = *r*, *y* > *w* > *s* *y* > *s*

62. (a) :

Equation of line = *x* + *y* = 41. If the (*x*, *y*) co-ordinates of the points are integer, their sum shall also be integers so that *x* + *y* = *k* (*k*, a variable) as we have to exclude points lying on the boundary of triangle; *k* can take all values from 1 to 40 only. *k* = 0 is also rejected as at *k* = 0 will give the point *A*; which can’t be taken.

Now, *x* + *y* = *k*, (*k* = 1, 2, 3, ... 40) with *k* = 40; *x* + *y* = 40; taking integral solutions.

We get points (1, 39), (2, 38); (3, 37) ...(39, 1) *i.e*. 39 points ;

*x* + *y* = 40 will be satisfied by 39 point.

Similarly *x* + *y* = 39 is satisfied by 38 points

*x* + *y* = 38 by 37 point

*x* + *y* = 3 by 2 points

*x* + *y* = 2 is satisfied by 1 point

*x* + *y* = 1 by no points.

So, the total number of all such points is:

39 + 38 + 37 + 36 + ... 3 + 2 + 1 = 

63. (d) : From the graph of (*y* − *x*) vs. (*y* + *x*), it is obvious that inclination is more than 45°.

Slope of line =  = tan (45° + θ);

 

By componendo-dividendo,   which is nothing but the slope of the line that shows the graph of *y* vs. *x*.

And as 0° < 45°, absolute value of less than 1.

  is negative and also, greater than 1.

The slope of the graph *y* vs. *x* must be negative and greater than 1. Accordingly, only option (d) satisfies.

You can also try by putting the values of (*y* + *x*) = 2(say) and

(*y* – *x*) = 4 (anything more than 2 for that matter). You can solve for values of *y* and *x* and cross check with the given options.

64. (b) : For the curves to intersect, log10 *x* = *x*–1

Thus, 

This is possible for only one value of *x* (2 < *x* < 3).

65. (d) : When we substitute values of *x* in the given curves

At *x* = –2 we get

*y*1 = –8 + 4 + 5 = 1

*y*2 = 4 – 2 + 5 = 7

Hence at *x* = –2 the curves do not intersect.

At *x* = 2, *y*1 = 17 and *y*2 = 11

At *x* = –1, *y*1 = 5 and *y*2 = 5

When *x* = 0, *y*1 = 5 and *y*2 = 5

And at *x* = 1, *y*1 = 7 and *y*2 = 7

Therefore, the two curves meet thrice when *x* = –1, 0 and 1.

66. (d) : Given *a*+ = max{*a*, 0}, we have to check the equality (*ab*)+ = (*a*+) (*b*+). It is clear that the equality holds true for all positive numbers. So, option (a) is true, but it is an incomplete set, as if one of *a* and *b* or both *a* and *b* is zero then also the equality holds true. Further, if one of *a* and *b* is negative and one of *a* and *b* is zero or positive, then also the equality holds ture. Further, if one of *a* and *b* is negative and one of *a* and *b* is zero or positive, then also the equality holds true.